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Analytic Approach in Solving Steady Laminar Flow of Fluid over a Stretching Sheet

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Abstract— We have considered the steady laminar flow over a linearly stretching sheet subjected to an order of chemical reaction. A similarity transformation is utilized to convert the governing nonlinear partial differential equations into ordinary differential equations. The local skin friction, rate of heat transfer and rate of mass transfer on the wall may calculate with the help of boundary conditions.

Keywords— Laminar flow, Chemical reaction, Similarity transformation, Heat and mass transfer, Stretching sheet

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1 Introduction

The heat and mass transfer in laminar boundary layer flow over a linearly stretching sheet have important applications in many fields of engineering. In addition, the diffusing species can be generated or absorbed due to some kind of chemical reaction with the ambient fluid. Mahapatra and Gupta (2002) studied the heat transfer in stagnation-point flow towards a stretching sheet. On the other hand, Afify (2004) analyzed the MHD free convective flow and mass transfer over a stretching sheet with homogeneous chemical reaction. Further, Alam and Ahammad (2011) investigated the effects of variable chemical reaction and variable electric conductivity on free convective heat and mass transfer flow over an inclined stretching sheet with variable heat and mass fluxes under the influence of Dufour and Soret effects. Ferdows and Qasem Al-Mdallal (2012) studied effects of order of chemical reaction on a boundary layer flow with heat and mass transfer over a linearly stretching sheet. Hayata et al. (2012) studied the unsteady three dimensional flow of couple stress fluid over a stretching surface with chemical reaction based on using homotopy analysis method. Singh (2012) investigated the MHD Flow with Viscous Dissipation and Chemical Reaction over a Stretching Porous Plate in Porous Medium. Makinde and Sibanda (2012) investigated the effects of chemical reaction on boundary layer flow past a vertical stretching surface in the presence of internal heat generation.

We have concerned with two dimensional steady, laminar flow of a fluid over a linearly stretching sheet. In this paper we have investigated analytically the effects of chemical reaction on the steady laminar two dimensional boundary layer flow and heat and mass transfer over a stretching sheet. The method of solution is based on the well-known similarity transformations.

2 Governing Equations

We have considered steady, laminar flow of a fluid over a stretching sheet. Again we have considered the stretched with a velocity proportional to x axis as shown below. We have assumed that the fluid far away from the sheet is at rest and at temperature T_∞ and concentration C_∞ . Further, the stretched sheet is kept at fixed temperature $T_w (< T_\infty)$ and concentration $C_w (< C_\infty)$.

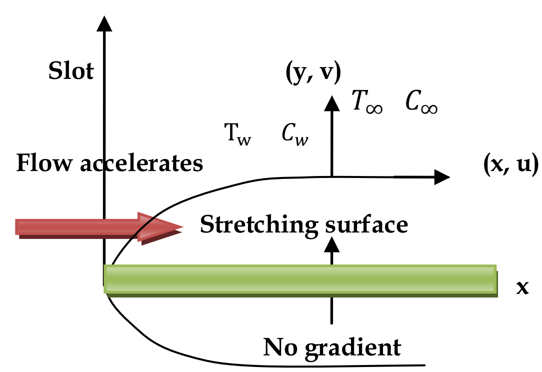

Figure.1 Boundary layer on a stretching surface

The equation governing the motion are :

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} \qquad (2)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{\upsilon}{C_p}\left(\frac{\partial u}{\partial y}\right)^2 \qquad (3)$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-k_1\,(C-C_\infty\,)^n \qquad (4)$$

3 BOUNDARY CONDITIONSS

$$u = u_0x,\ v = 0,\ T = T_w\ ,\ C = C_w \quad \text{at } y = 0$$

$$u\rightarrow 0,\ T\rightarrow T_w\,,\ C\rightarrow C_w\ ,\ \text{as } y\rightarrow \infty \qquad (5)$$

4 NOMENCLATURE

u → velocity component in the x direction
v → velocity component in the y direction
T → the fluid temperature in the boundary layer
C → concentration of the fluid
ϑ → kinematic viscosity
α → thermal diffusivity
C_p → specific heat at constant pressure
k_1 → constant of first-order chemical reaction rate
D → effective diffusion coefficient
η → similarity variable
f → dimensionless stream function
θ → dimensionless temperature
ϕ → dimensionless concentration
ψ → stream function

5 MATHEMATICAL FORMULATION

In order to solve Equations (1)-(5), we introduce the following similarity transformation:

$$\eta(x,y)=y\sqrt{\frac{u_0}{\upsilon}} \qquad (6)$$

$$\Psi(x,y)=\ f(\eta)\,x\sqrt{\upsilon u_0} \qquad (7)$$

$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty} \qquad (8)$$

$$\varphi(\eta)=\frac{C-C_\infty}{C_w-C_\infty} \qquad (9)$$

We define the stream function defined as

$$u=\frac{\partial\Psi}{\partial y},\ v=-\frac{\partial\Psi}{\partial x} \qquad (10)$$

Now we get

$$u=\frac{\partial\Psi}{\partial y}$$
$$=\frac{\partial}{\partial y}\{f(\eta)\,x\sqrt{\upsilon u_0}\}$$
$$=x\sqrt{\upsilon u_0}\,f'(\eta)\,\frac{\partial\eta}{\partial y}$$
$$=x\sqrt{\upsilon u_0}\,f'(\eta)\sqrt{\frac{u_0}{\upsilon}}$$
$$=u_0xf'(\eta)\,.$$

And

$$v=-\frac{\partial\Psi}{\partial x}$$
$$=-\frac{\partial}{\partial x}\{f(\eta)\,x\sqrt{\upsilon u_0}\}$$
$$=-f(\eta)\sqrt{\upsilon u_0}-f'(\eta)\,x\sqrt{\upsilon u_0}\,.0$$
$$=-f(\eta)\sqrt{\upsilon u_0}$$

Also

$$\frac{\partial u}{\partial x}=\frac{\partial}{\partial x}\{u_0x\,f'(\eta)\}$$
$$=u_0f'(\eta)$$
$$\frac{\partial u}{\partial y}=\frac{\partial}{\partial y}\{u_0x\,f'(\eta)\}$$
$$=u_0xf'(\eta)\sqrt{\frac{u_0}{\upsilon}}\ .$$

And

$$\frac{\partial^2 u}{\partial y^2}=\frac{\partial}{\partial y}\{u_0x\,f'(\eta)\sqrt{\frac{u_0}{\upsilon}}\}$$
$$=u_0x\,f''(\eta)\sqrt{\frac{u_0}{\upsilon}}\sqrt{\frac{u_0}{\upsilon}}$$
$$=u_0x\,f''(\eta)\frac{u_0}{\upsilon}\ .$$

Putting these values in equation (2), we get

$$\{u_0xf'(\eta)\}\{u_0f'(\eta)\}+\{-f(\eta)\sqrt{\upsilon u_0}\}\{u_0x\,f'(\eta)\sqrt{\frac{u_0}{\upsilon}}\}=u_0{}^2x\,f'''(\eta)$$

$$or,\quad u_0{}^2x\,\{f'(\eta)\}^2-u_0{}^2x\,f(\eta)f''(\eta)=u_0{}^2x\,f'''(\eta)$$

$$or,\quad \{f'(\eta)\}^2-f(\eta)f''(\eta)=\ f'''(\eta)$$

$$or,\quad f'''(\eta)+\ f(\eta)f''(\eta)-\{f'(\eta)\}^2=0\ . \qquad (11)$$

Again we get

$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}$$

$$or,\quad T-T_\infty=\theta(\eta)(T_w\ -T_\infty\,)$$

$$or,\quad T=\ T_\infty+\theta(\eta)(T_w\ -\ T_\infty\,)\,. \qquad (12)$$

Now

$$\frac{\partial T}{\partial x}=\frac{\partial}{\partial x}\{T_\infty+\theta(\eta)(T_w-T_\infty\,)\}\ =\ 0$$

$$\frac{\partial T}{\partial y}=\frac{\partial}{\partial y}\{\,T_\infty+\theta(\eta)(T_w-T_\infty\,)\,\}$$
$$=\theta'(\eta)\sqrt{\frac{u_0}{\upsilon}}(T_w-T_\infty\,)$$

and

$$\frac{\partial^2 T}{\partial y^2}=\frac{\partial}{\partial y}\{\theta'(\eta)\sqrt{\frac{u_0}{\upsilon}}(T_w-T_\infty\,)\}$$

$$= \theta''(\eta)\frac{u_0}{\upsilon}(T_w - T_\infty)$$

$$\frac{\partial u}{\partial y} = u_0 x\, f'(\eta)\sqrt{\frac{u_0}{\upsilon}}$$

$$= u\, f''(\eta)\sqrt{\frac{u_0}{\upsilon}}\,.$$

Now, putting these values in equation (3), we get

$$-f(\eta)\sqrt{\upsilon u_0}\,\theta'(\eta)\sqrt{\frac{u_0}{\upsilon}}(T_w - T_\infty) =$$

$$\alpha\theta''(\eta)\frac{u_0}{\upsilon}(T_w - T_\infty) + \frac{\upsilon}{C_p}\left\{u\, f''(\eta)\sqrt{\frac{u_0}{\upsilon}}\right\}^2$$

$$or,\ -u_0 f(\eta)\theta'(\eta)(T_w - T_\infty) =$$

$$\alpha\,\theta''(\eta)\frac{u_0}{\upsilon}(T_w - T_\infty) + u_0\frac{u^2}{C_p}\{f''(\eta)\}^2$$

$$or, -f(\eta)\theta'(\eta)(T_w - T_\infty) = \theta''(\eta)\frac{\alpha}{\upsilon}(T_w - T_\infty)$$

$$+ \frac{u^2}{C_p}\{f''(\eta)\}^2$$

$$or,\ -f(\eta)\theta'(\eta) = \theta''(\eta)\frac{\alpha}{\upsilon} + \frac{u^2}{C_p(T_w - T_\infty)}\{f''(\eta)\}^2$$

$$or,\ \theta''(\eta)\frac{\alpha}{\upsilon} + f(\eta)\theta'(\eta) + \frac{u^2}{C_p(T_w - T_\infty)}\{f''(\eta)\}^2 = 0$$

$$or,\ \theta''(\eta) + \frac{\upsilon}{\alpha}\left[f(\eta)\theta'(\eta) + \frac{u^2}{C_p(T_w - T_\infty)}\{f''(\eta)\}^2\right] = 0$$

$$or,\ \theta''(\eta) + Pr[\,f(\eta)\theta'(\eta) + Ec\{f''(\eta)\}^2] = 0 \quad (13)$$

where,

$$Pr = \frac{\upsilon}{\alpha}\ .$$

And

$$Ec = \frac{u^2}{C_p(T_w - T_\infty)}\ .$$

Again we get

$$\varphi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}$$

$$or,\quad C - C_\infty = \varphi(\eta)(C_w - C_\infty)$$

$$or,\quad C = C_\infty + \varphi(\eta)(C_w - C_\infty)\,. \quad (14)$$

Now

$$\frac{\partial C}{\partial x} = \frac{\partial}{\partial x}\{C_\infty + \varphi(\eta)(C_w - C_\infty)\} = 0$$

$$\frac{\partial C}{\partial y} = \frac{\partial}{\partial y}\{C_\infty + \varphi(\eta)(C_w - C_\infty)\}$$

$$= \varphi'(\eta)\sqrt{\frac{u_0}{\upsilon}}(C_w - C_\infty)$$

And

$$\frac{\partial^2 C}{\partial y^2} = \frac{\partial}{\partial y}\left\{\varphi'(\eta)\sqrt{\frac{u_0}{\upsilon}}(C_w - C_\infty)\right\}$$

$$= \varphi''(\eta)\sqrt{\frac{u_0}{\upsilon}}\sqrt{\frac{u_0}{\upsilon}}(C_w - C_\infty)$$

$$= \varphi''(\eta)(C_w - C_\infty)\frac{u_0}{\upsilon}\,.$$

Putting these values in equation (4), we get

$$-f(\eta)\sqrt{\upsilon u_0}\,\varphi'(\eta)\sqrt{\frac{u_0}{\upsilon}}(C_w - C_\infty) = D\varphi''(\eta)$$

$$.(C_w - C_\infty)\frac{u_0}{\upsilon} - k_1\{\varphi(\eta)(C_w - C_\infty)\}^n$$

$$or, -f(\eta)\varphi'(\eta)u_0(C_w - C_\infty) = D\varphi''(\eta).$$

$$(C_w - C_\infty)\frac{u_0}{\upsilon} - \varphi^n(\eta)(C_w - C_\infty)^n$$

$$or, -f(\eta)\varphi'(\eta) = D\varphi''(\eta)\frac{1}{\upsilon} - k_1\ \varphi^n(\eta)\frac{(C_w - C_\infty)^n}{u_0(C_w - C_\infty)}$$

$$or, -f(\eta)\varphi'(\eta) = \varphi''(\eta)\frac{D}{\upsilon} - k_1\varphi^n(\eta)\frac{(C_w - C_\infty)^{n-1}}{u_0}$$

$$or,\ \varphi''(\eta) + \frac{\upsilon}{D}\left[f(\eta)\varphi'(\eta) - k_1\varphi^n(\eta)\frac{(C_w - C_\infty)^{n-1}}{u_0}\right] = 0$$

$$or,\ \varphi''(\eta) + Sc[\,f(\eta)\varphi'(\eta) - Cr\ \varphi^n(\eta)\,] = 0 \quad (15)$$

where

$$Sc = \frac{\upsilon}{D}\,.$$

And

$$Cr = k_1\frac{(C_w - C_\infty)^{n-1}}{u_0}\,.$$

Consequently, equations (2)-(4) and the boundary conditions (5) can be written in the following form,

$$f'''(\eta) + f(\eta)f''(\eta) - \{f'(\eta)\}^2 = 0 \quad (16)$$

$$\theta''(\eta) + Pr[\,f(\eta)\theta'(\eta) + Ec\{f''(\eta)\}^2] = 0 \quad (17)$$

$$\varphi''(\eta) + Sc[\,f(\eta)\varphi'(\eta) - Cr\,\varphi^n(\eta)\,] = 0 \quad (18)$$

subject to the boundary conditions

$$f(0) = 0, f'(0) = 1, \theta(0) = 1, \varphi(0) = 1$$

$$f'(\infty) = 0, \theta(\infty) = 0, \varphi(\infty) = 0\,. \quad (19)$$

where ,

$Pr = \frac{\upsilon}{\alpha}$ represents Prandtl number,

$Ec = \frac{u^2}{C_p(T_w - T_\infty)}$ represents Eckert number,

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$Sc = \frac{\upsilon}{D}$ represents Schmidt number,

$\mathrm{Cr} = k_1 \frac{(C_w - C_\infty)^{n-1}}{u_0}$ represents the chemical reaction parameter.

From the equations (16), (17) and (18) we can find out the local skin-friction coefficient, $-f''(0)$, rate of heat transfers, $-\theta'(0)$ and rate of mass transfers, $-\varphi'(0)$ as

$$f''(0) = \frac{1}{u_0\sqrt{x}} (Re)^{-1/2} \left(\frac{\partial u}{\partial y}\right)_{y=0} \tag{20}$$

$$\theta'(0) = \frac{\sqrt{x}}{(T_w - T_\infty)} (Re)^{-1/2} \left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{21}$$

$$\varphi'(0) = \frac{\sqrt{x}}{(C_w - C_\infty)} (Re)^{-1/2} \left(\frac{\partial C}{\partial y}\right)_{y=0} \tag{22}$$

where, $\mathrm{Re} = \frac{u_0 x}{\upsilon}$ is the Reynolds number.

6 Conclusion

We have considered steady laminar flow over a stretching sheet in the present of chemical reaction. By using some suitable transformations we have reduced the partial differential equation into ordinary differential equation. Finally we have derived a set of equation from which we can find local skin-friction coefficient, rate of heat transfer and rate of mass transfer in terms of Reynolds number.

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